

# “I Could Have Used This When I Was in School”

## Stories from the Maple™ - Deprived

Over my long career at Maplesoft, I have shown Maple™ to a lot of new employees. And whether they have a PhD in math or dropped math in high school as soon as they could get away with it, there is almost always a moment where the person stops, stares, and says some variation of “I could have used that when I was in school.”

So I decided to collect some of these stories as a way to show off the many different ways Maple can make a difference to students. Like the students in your own classes, the people I talked to had a variety of backgrounds, mathematical abilities, and interest in math. These stories come from R&D, sales and

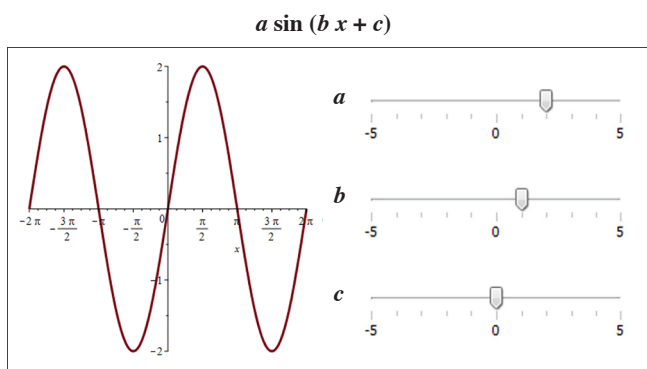
marketing, and customer service. They come from people who use Maple every day and those who have never even installed it. They come from people with advanced math degrees, and from people who hated math in school and are astonished to find themselves working for a math software company now.

*Note:* I promised I wouldn't use names because some people really hate that, but these are all stories from current or former Maplesoft employees. I helped write some of their stories, based on our conversations, but everyone has approved their quote.

## Understanding

The first story is mine.

I've always liked abstract math – the more abstract, the better (category theory was very satisfying). It never bothered me in the slightest that there was no physical interpretation to most of what I learned. In fact, I preferred it that way. I'm simply not a very visual person, in defiance of the math-ability stereotypes, and trying to visualize things can sometimes feel like I'm fighting my brain's natural thought patterns. I can do it, but it's a lot of work.



So I experienced a powerful “I could have used that!” moment the first time I saw a version of this interactive “slider plot”.

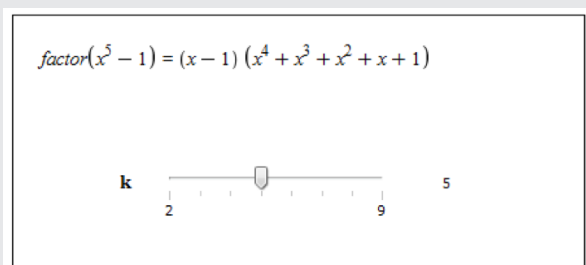
I had a flashback to sitting at my desk, calculating sample data points on my calculator and sketching tiny little graphs in the margin of my notebook, so I could figure out if the parameters in my trig function changed the amplitude, the period, or the phase shift, and how. I had no problems with the general concept, but I had a harder time remembering which parameter did what. Unfortunately, it took so long to create the sample graphs that I couldn't do enough examples to get to the point where I didn't have to think about it anymore. I'm convinced that a few minutes of playing with that slider plot would have really helped.

Of course, unlike me, some people need the visuals:

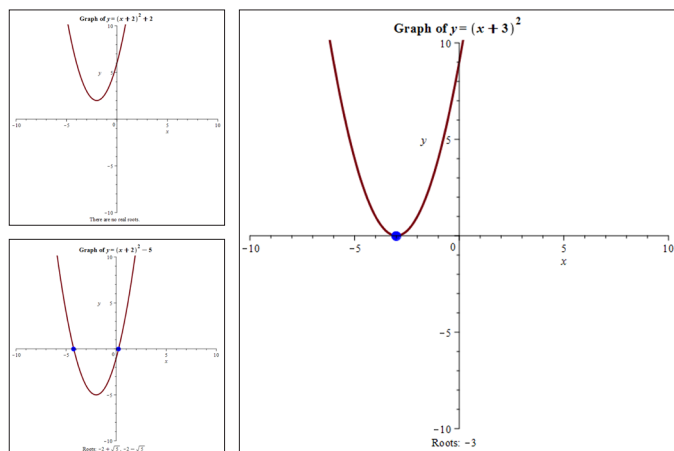
*“I took math all through high school, but I never considered myself to be good at it. I'm a super visual person, and so much of what I was taught felt like a bunch of meaningless symbols on the page. I need to see it to understand it. After I started working here, I remember sitting in on a demo, and someone showed a Math App about something I remember NEVER made sense to me when I was in school. Suddenly, for the first time, I got it.”*

### More on... Exploring Plots and Expressions

The Explore facility in Maple lets you create parameterized plots and expressions for just about anything. You use sliders to change the value of the parameters, and the result is displayed immediately. You can create these interactive mini-applications in a single step, using the interactive Exploration Assistant or through the Explore command. The parameterized sine plot shown shown on the left was created using Explore. Explore can also be used with expressions. The following example explores the factors of  $(x^k - 1)$  for different values of  $k$ .



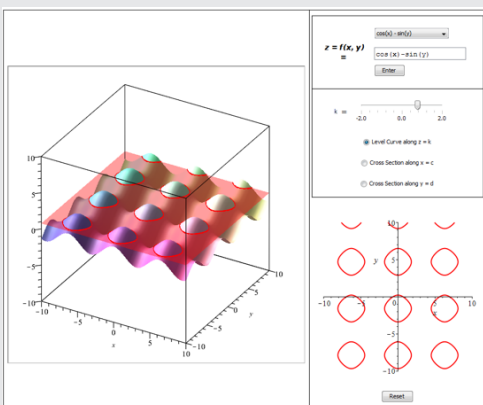
The Math App they were referring to was examining the root of quadratic equations:



This App shows how a quadratic can have zero, one, or two roots, depending on how many places the curve crosses the x axis. When they saw the curve move from above the axis, to just touching, to below the axis where it crossed in two places, something finally clicked.

### More on... Math Apps

Maple contains hundreds of interactive Math Apps, like the Roots of a Quadratic app above, which are designed to help students visualize mathematical concepts. They cover a wide range of topics, such as functions and relations, calculus, vectors and planes, discrete math, physics, finance, probability, and more. Here's another example, which explores level curves and cross sections.



There is no doubt that visualization helps with understanding many mathematical concepts. But sometimes the understanding the student is looking for is not some new math concept, but simply “Where did I go wrong?”

*“One of my biggest frustrations with math in high school was the times I would do a problem, look at the back of the book, and see that I got the wrong answer. And I didn't have a clue why. All I knew was that my answer was wrong, with no indication of why or what I could do about it. There were many times I felt like I was banging my head against a wall. If I had had Maple, I could have used it to check my steps, and figure out where I went wrong. Instead of being incredibly frustrated, I would have had a tool to help me figure things out.”*

### More on... Context-Sensitive Menus

Whether a student is checking their final answer, or checking each step of their derivation to determine where they made a mistake, they are likely using Maple's Clickable Math™ tools to do it. Probably the most important Clickable Math tools in Maple are the context sensitive menus for performing mathematical operations. These menus change according to the mathematical expression, so only relevant choices are displayed. Using these menus, students can solve for  $x$ , plot an expression, integrate with respect to  $t$ , find a determinant, manipulate an equation, factor a polynomial, evaluate their expression at a point, and much more, all at the click of a button.

For the most confident students, the first question isn't necessarily "where did I go wrong", but "did I actually go wrong?"

*"When your answer doesn't match the answer in the back of the book, and you know the book is sometimes wrong, you don't know who to believe. If I could have checked the answer using Maple, it would have been clear. Either the book was wrong, and I could relax, or I was wrong, and I'd know for sure that I had made a mistake, and that I wouldn't be wasting my time by going over the question again."*

## Skills Development

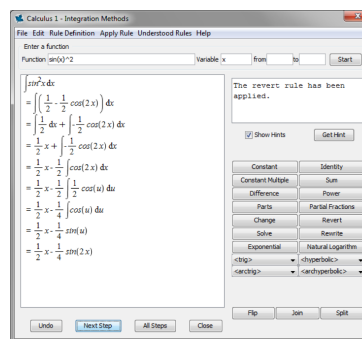
Sometimes understanding comes from the "aha" moment, but most often it comes through practice, and lots of it. But opportunities for meaningful practice can be hard to come by.

*"I always found there were too few examples to try in the text book and/or course notes. Things like matrix multiplication or solving integrals/derivatives take a lot of practice. The textbook would tend to have one question for each "gotcha" -- you had to work through that one with the answer in front of you, and were never sure if you understood the concept,*

*or just memorized that particular example. Making up your own examples is pointless without having a way to verify your answer. Maple would have been a great tool to (a) generate example problems (ones that weren't so complex that they would take you down a rat hole), and (b) check that your answer is correct."*

Integration practice, in particular, came up more than once in the conversations I had:

*"In Maple, there is an Integration Methods Tutor that I really wish I had at school back in my day. Specifically, integration by substitution is often a matter of trial and error, and it requires experience to find a good substitution for an antiderivative. The advantage of the tutor is you can quickly try out different substitutions and check the complexity of the result without too much manual labour, compared with using pen and paper."*



## More on... Tutors and Student Packages

Learning math is not just about getting the answer, so in addition to its computation abilities, Maple also provides specialized student packages that offer focused learning environments in which students can explore and reinforce fundamental concepts in the same way their instructor does in class. Student packages are available for calculus, precalculus, linear algebra, statistics, vector calculus, multivariate calculus, numerical analysis, and more.

Many of these student packages include interactive tutors for exploring, understanding, visualizing, and practicing concepts. The Integration Methods Tutor is an example of a tutor that provides guided step-by-step problem solving. Maple also provides step-by-step tutors for differentiation, limits, inverting matrices, Gaussian elimination, linear system solving, and others.

## Effective Use of Learning Time

Students are busy, so it's no surprise that many of the stories people told me revolved around ways that Maple could have saved them time and helped them work more efficiently.

*"One of the first tasks I undertook when learning Maple was to solve problems in perturbation theory, having just studied the subject in the months prior. Solving such problems can be very tedious, especially for examples with multiple time scales and singular perturbations. To account for a large number of terms, very long expressions, and successive steps, just to solve for the appropriate parameter values can be a delicate and long feat by hand. As a result, I had to spend a lot of time on the mechanics of finding the solution long after I had understood how to solve it. Maple would have helped me reach the solution much more quickly, without changing how much I learned from the experience."*

Sometimes the issue is that students are forced to use an unnatural approach to solve a mathematical problem.

### More on... Symbolics

Maple is a symbolic computation system, which means it is capable of manipulating symbols, such as  $x$  and  $y$ , not just numbers. As a result, problem solving in Maple is a lot more natural than numeric computation systems like MATLAB, because the student can express a problem the same way they would on paper, and the results are expressed as proper mathematical expressions, not just numbers.

*"I used MATLAB® in 3rd year. When it came to solving differential equations, it took a while to get used to the artificial way you had to create sample time points and sample value points to feed into the ode45 solver. Switching to Maple later, and getting to work with equations instead, felt way more natural and intuitive."*

Similarly, it is frustrating to be forced to perform a repetitive task manually when a better method is available:

*"As a chemical engineering student, I spent far too much time getting thermodynamic and fluid data by eyeballing printed charts or interpolating between numeric data in tables. With Maple, that data can be extracted with a function call. That's 1000s of times faster than getting data by hand! I can also use that data in a Maple calculation with units checking. That would have saved me so much time, and helped me to learn more about the engineering principles involved."*

### More on... the Math Engine

Maple includes over 5,000 computational functions covering virtually every area of mathematics, as well as many specialized applications, including:

Calculus	Linear Algebra
Differential Equations	Algebra
Combinatorics	Statistics
Physics	Number Theory
Scientific Constants	Signal Processing
Linear Programming	Differential Algebra
Dynamic Systems	Lie Symmetries
Computational Geometry	Finance

...and many more. Maple also includes a programming language that was designed for mathematics, which can be used to customize and extend Maple's abilities.

Maple can certainly help students solve problems faster than by hand, but Maple can also help save time over the entire assignment.

*"I was fortunate to be able to start using Maple partway through my engineering degree. It made a big difference in how I did my assignments, which typically consisted of pages of explanations, diagrams, and calculations. Without Maple, finding a mistake half way down the page resulted in a LOT of erasing and redoing. When I switched to using Maple, I could do the entire assignment in a Maple document, and be just as productive within Maple as I could with paper and pen. Better, I didn't have to worry about calculation errors, and if I found a mistake in the logic, I could make changes without using up half an eraser. When I was done, I exported the results to a PDF and handed it in. Since the focus in my courses was on knowing what to do, no one objected to my using Maple this way. In fact, some instructors preferred it because then they didn't need to decipher my handwriting."*

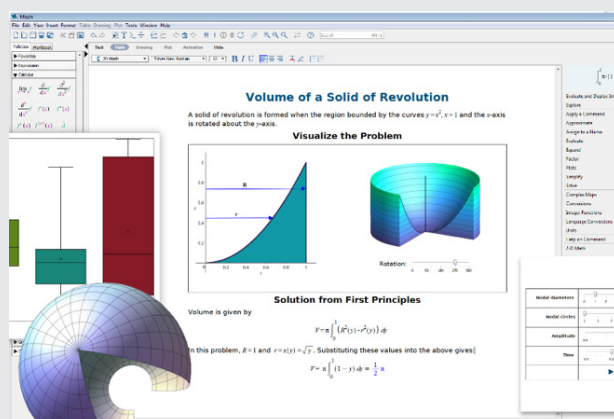
And it's not just the students' handwriting that can be problematic.

*"One incident that comes to mind is this: In an advanced math course I took many years ago, the professor's handwriting was terrible. When my friends and I compared our notes later, we found, to our dismay, that they did not match. In several of the equations we'd copied down into our notebooks, the Greek variables were different, with phi and psi mixed up, for example. Since a typical lecture incorporated almost the entire Greek alphabet, our carefully copied notes were essentially useless. The conclusion: instructors with terrible handwriting should use Maple documents to present their lectures."*

Underlying these anecdotes is the feeling that, while a student may be perfectly willing to pour hours of work into a course, they want to feel like that time isn't wasted. Performing a task in a slow, inefficient, or unnatural way takes up time that could be better spent on activities that have a greater impact on their learning, such as reviewing the day's lecture notes, trying additional practice problem, checking over their assignment, or even getting a reasonable amount of sleep.

## More on... Technical Documents

Maple provides a technical document interface that allows you to combine computations, text, plots, images, and more in a single document. Documents are live, so if you change your problem, you can re-execute your document and all the computations and plots are updated automatically. Maple documents can also contain components such as buttons and sliders, which turn the document into an interactive application.



## Confidence

While it wasn't necessarily what these contributors were thinking at the time, many of these stories can be viewed as "Maple would have given me more confidence": Confidence that my answer is correct. Confidence that, if my answer is not correct, I will be able to fix it. Confidence that I'm not wasting my time. Confidence that comes through practice. Confidence that I really understand.

Maple helps students understand new concepts, build up their skills through practice, check their answers so they know if they are right, and if they are not, Maple gives them tools they can use to discover where they've gone wrong. All these uses of Maple contribute directly to students succeeding in their courses. But it's important to note that these activities also help develop a student's confidence in their mathematical abilities. Since it's well established that confidence, or lack thereof, makes an especially big difference to student success in math courses, the fact that Maple can help develop a student's confidence is far more important than any particular feature or example.

I'll end with another story from my own experience.

My high school, unlike any other school I've ever come across, told students who were struggling with math to take Calculus if they needed a single math credit in their final year to get into their chosen post-secondary program. When I took high school Calculus, about a quarter of the class was taking the course for the second time, and still struggling. The students like me, who were good at math, coasted through the course

without ever having to think too deeply about what we were doing (actually, we played a lot of Euchre during the abundant work periods). Combined with my natural tendency not to visualize anything if I wasn't forced to, this experience left me ill-prepared for my extra-advanced Calculus course in first-year university, where the professor started every class by putting a poorly behaved function on the blackboard, and asking us to figure out what it looked like. He kept saying things like "can you see that, as  $x$  gets close to zero, this term will dominate?", "can you see that the curve will bend like this at about this point?", "can you see that...?". At that time, I still had to think for a moment

to be sure I wasn't getting logarithms and exponentials mixed up, so my answer to "can you see that...?" was invariably "no".

So then I had two problems. First, I was genuinely less

prepared for this course than many of my classmates, and second, my confidence in my ability to succeed in that advanced class was quickly destroyed, which in turn started to seriously affect my performance in the course as a whole. I got through it, eventually, but it was a real struggle for most of that term.

If I had had Maple\*, I could have used a systematic approach to build up my plotting intuition – something I hadn't ever realized I needed and was missing. I could have tried plotting example after example of basic functions, immediately seeing the effect of varying parameters, and combining expressions like we did in class. I could have guessed what they would look like and then checked how my answer compared, until I no longer had to think so hard about every little thing. That understanding would have definitely helped. But what I

“ But what I think would have made an even bigger difference is that I would have felt a whole lot more confident in my own abilities... ”

*\* To be fair, technically I did have access to Maple around that time, though I didn't realize it. But ASCII-rendered plots scrolling past the top of green CRT terminal would not have had the same illuminating effect.*



think would have made an even bigger difference is that I would have felt a whole lot more confident in my own abilities relative to my classmates, and hearing "can you see that...?" would have stopped filling me with such a sense of dread *even if the answer was still "no"*. (Which it probably would have been often enough - he picked some truly pathological functions!)

The people in this article did not have access to today's Maple throughout their education, but they all felt that having Maple would have made their student experience better. Your students are fortunate, because they can have these benefits today, instead of looking back years later and saying "I could have used that". However, if your students don't already have access to Maple, they likely need your help, or at least your encouragement, to get this better experience. It's something to think about.

## About the Author

Eithne Murray has been around Maple for much of her academic and professional life, such as using Maple for her Master's thesis in computational number theory, helping develop Maple's combinatorial structures package, and contributing to a textbook on using Maple in discrete mathematics. She has worked for symbolic computation research labs in France and Canada, and has held a variety of roles at Maplesoft in both R&D and marketing. She still isn't particularly fond of visualization, but since the widespread availability of GPS technology, she gets lost much less often.